Hybrid Anticipatory Networks

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Abstract. This paper presents a theory of hybrid anticipatory networks that generalizes earlier models of consequence anticipation in multicriteria decision problems. We assume that the decision maker takes into account the anticipated outcomes of future decision problems linked by the causal relations with the present decision problem. This can be represented by a multigraph, where decision problems are modeled as nodes linked causally and by one or more additional anticipation relations. These types of multigraphs are termed anticipatory networks. Hybrid anticipatory systems may contain additional models of random and non-cooperative game decisions. Constructive solution methods for decision problems modeled by anticipatory networks are discussed as well. Further, we present a generalization of hybrid anticipatory networks, known as superanticipatory systems. In the final section we discuss some of their applications in the design of decision-making rules in autonomous robotic systems and in filtering technology development scenarios.

Keywords: Anticipatory networks, decision theory, multicriteria optimization, analysis of consequences, superanticipatory systems.

1 Introduction

The introduction of anticipatory networks as models of future consequences in a decision-making process was inspired by the idea formulated in [6] as "*To use anticipated future consequences of a decision as a source of additional preference information in multicriteria decision problems*". The exploration of such anticipatory feedback is possible owing to the following assumptions:

1. A decision maker is responsible for solving a decision problem which corresponds to the starting node of the anticipatory network.
2. There exist estimates (forecasts or foresight scenarios) of future decision problem formulations, their solution rules, decision makers' preferences and of the relations binding their anticipated outcomes with the current problem.
3. The decision maker knows the causal structure of future decision problems that are modeled by the other nodes of the network, in particular the way in which problem parameters are influenced by solutions to preceding decision problems.

The first and third assumptions allow us to model the impact of a decision to-be-made on any subsequent problem in the network. The second assumption is a basis for

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defining anticipatory relations, which describe the *present* interests in the *future* outcomes that depend on *present* decisions.

The usual approach used in decision theory is to model the consequences of the decision with just one value, called utility [1,2,6]. The utility refers implicitly to the future, and in multicriteria decision problems it indicates how the values of multiple optimization criteria should be assessed. The assumption that there may exist multiple measures of utility led in [6] to an anticipatory model of consequences in a sequence of multicriteria decision problems. This was expanded in a series of later papers [8,10] to a theory of new networks, called *optimizer networks,* which model the optimization problems and their temporal environment. An *optimizer* O acts on a set of feasible decisions U and on a preference structure P and selects a subset $X \subset U$ according to P and to a fixed set of optimization criteria F that are characteristic for this optimizer. The optimization problems modeled as the optimizers have the form

$$(F: U \rightarrow E) \rightarrow min(P) \tag{1}$$

where P is a general preference structure in the sense of Yu and Leitmann [12] defined as $P:=\{\pi(u) \subset U: u \in U\}$ and such that if $v \in \pi(u)$ and $w \in \pi(v)$ then $w \in \pi(u)$. Usually E is a vector space with a partial order $\leq_\theta$ introduced by a convex cone θ, and

$$\pi(u):= \pi(u,\theta)=\{v \in U: F(v) \leq_\theta F(u)\}.$$

A *free optimizer* O may select any solution u_0 from U that is nondominated with respect to P and F in (1), i.e. u_0 belongs to the set

$$\Pi(U,F,P):=\{u \in U: [\ \forall v \in U: F(v) \leq_\theta F(u) \Rightarrow v=u]\}. \tag{2}$$

O is then uniquely characterized by U, F, and P and may be denoted as a 3-tuple $O:=(U,F,P)$. If the admissible solution set in an optimizer O may be different from $\Pi(U,F,P)$ and equal to $X \subset \Pi(U,F,P)$, we will denote it as $O:=(U,F,P,X)$. X will be interpreted as the required solution set that defines the optimization principle applied to F in the problem (1).

In addition to their optimizing capabilities, the optimizers may influence each other, forming thus networks with essentially new properties compared to the former theory of linked multicriteria decision problems [6,10]. In particular, in feed-forward networks of optimizers, constraints and preference structures in some optimizers are causally linked to the solutions of other problems and may depend on their preference structures. Thus, in a network of optimizers, the parameters of actual instances of optimization problems to-be-solved depend on the results of solving other problems in the network.

An influence relation r represented by the network of optimizers may be defined as

$$O_1\ r\ O_2 \Leftrightarrow \exists(\varphi: X_1 \rightarrow 2^{U_2}) \text{ such that } X_2=\varphi(X_1), \tag{3}$$

where $O_1:=(U_1,F_1,P_1,X_1)$, $O_2:=(U_2,F_2,P_2,X_2)$, and φ is the multifunction that defines the influence of the solutions to O_1 on the set of admissible decisions in O_2. Influence relations linking preference structures may be defined analogously, using multifunctions from X_1 to the family of preference structures in U_2 to modify P_2. If r is acyclic it will be termed a *causal influence relation*. The term *causal network* will refer to the graph of a causal influence relation. In a causal network of optimizers the function φ influences the constraints in O_i by outputs from the problems preceding O_i in r.

This paper presents a theory of hybrid anticipatory networks that generalizes the above outlined model of consequence anticipation in multicriteria optimization. Motivated by real-life problems, apart from optimizers, the hybrid networks may contain nodes related to non-cooperative game solutions and those describing random decisions. In addition, the situations where the decision is pre-determined, e.g. by optimizing a single criterion on a fixed set, or by using a deterministic decision-making algorithm, are modeled as separate "algorithmic decision" units in the causal network.

To sum up, similarly as in [6,8,10] we assume that while making the decision at the starting node of the anticipatory network, the decision maker takes into account forecasts concerning the parameters of future decision problems, anticipation of future decision makers' behavior, the forecasted causal dependence relations linking the parameters of decision problems in the network, and the anticipatory relations pointing out which future outcomes are relevant to decision making at the specified causally-preceding problems. There exists an additional preference structure at the starting problem O_0 that specifies the solution subsets to future problems that are regarded as required (or desired in a relaxed formulation of the problem) by the decision maker at O_0.

Moreover, it is assumed that an additional preference structure concerning future decision problems may occur at other nodes of the network as well. This led us to the introduction of so-called *superanticipatory systems* (cf. Sec. 3 and [8]). Recall that a system is *anticipatory* in the sense of Rosen [5,4] iff it contains a model of itself and of the outer environment, and its future extrapolation. By definition, a superanticipatory system is an anticipatory system that contains a future model of at least one other anticipatory input-output system whose outcomes may influence decisions at a causally-preceding problem by an anticipatory feedback relation. It will be observed that most anticipatory networks are superanticipatory systems because decisions at future nodes can be based on similar anticipatory principles as those applied at the current node.

The next Secs. 2 and 3 will show the basic properties of anticipatory networks and propose a method of solving the corresponding decision problems, network transformations and computing, so that the additional preferences concerning the required (or desired) future decisions are taken into account at the starting node.

2 Basic Properties and Structure of Hybrid Anticipatory Networks

As pointed out in the preceding section, we will construct anticipatory networks with nodes modeling different types of future decision problems. Apart from optimization problems we can also model the choice of a mixed strategy in games with conflicts that may eventually lead to Nash equilibria, subset selection problems, pre-determined, random or irrational decisions. Hybrid anticipatory networks may contain nodes of all types, but their structure is similar to networks of optimizers as the nodes are connected by edges modeling causal and anticipatory relations. All nodes in an anticipatory network will be termed *decision units* (DU), while optimizers and non-cooperative game units will be additionally termed *controllable*.

Similar to the free optimizers defined in Sec.1, the decision units of all kinds produce an output decision based on the inputs fed by units preceding them in the causal order. In Fig.1 below we present the schemes of the most important decision problems that can be modeled as nodes in hybrid anticipatory networks.

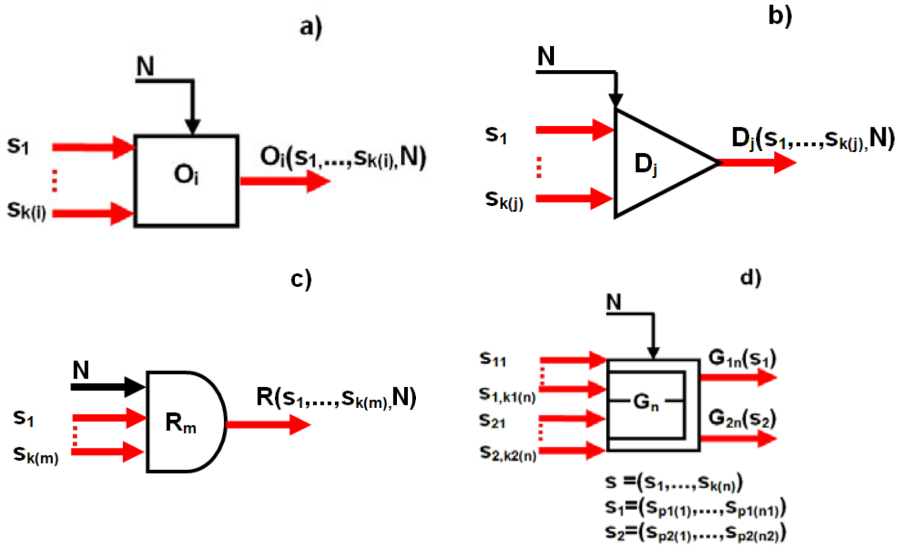

Fig. 1. The basic decision units that may occur in a hybrid anticipatory network: simple box (a) - multiple-input optimizer with output possibly influenced by the states of nature *N*, triangle (b) - the pre-determined algorithmic decision unit, where the decision may additionally depend on the states of nature *N*, rounded box (c) – random decision is selected based on known inputs and an output distribution function, subdivided box (d) – 2-player non-cooperative game unit

The inputs in all units presented in Fig.1 depend on the outputs from other units except the initial node, which has no inputs unless it is influenced by the external environment (the nature) *N*. This dependence may have the form of a multifunction φ that defines the influence relation *r* (3), or another influence relation that e.g. modifies the preference structure in an optimizer or the information set in a game unit. The influence relations are characteristics of edges that link the DUs, but they always depend on the output from their starting node. Every decision unit has two functions:

(i) first, it transforms (aggregates) the input signals into the mapping that modifies the parameters of the decision problem to be solved by this DU; this transformation is a characteristic feature of a particular DU;

(ii) second, it solves its decision problem with modified parameters and produces the output, which can be identified with the solution of the DU's characteristic problem; the output is unique except the game units, where the number of outputs equals the number of players.

For instance, the aggregation of input signals represented by multifunctions $\varphi_1,\ldots,\varphi_k$ that restrict the choice of an admissible decision at a decision unit O by imposing additional constraints (3) may be defined as an intersection $\varphi=\varphi_1\cap\ldots\cap\varphi_k$, i.e. if U is the decision set at O and the outputs at the decision units causally preceding O are $x_1,\ldots,x_k$ then

$$\varphi(x_1,\ldots,x_k)=\varphi_1(x_1)\cap\ldots\cap\varphi_k(x_k)\subset U.$$

In general, aggregations can be defined by arbitrary Boolean and algebraic operations, depending on the modeling purposes.

We will say that a decision unit O is *active*, if a decision-maker that performs a free choice [9] can be associated with O. Otherwise it is termed *passive*. The operations of the decision units are summarized in the following Tab.1.

Table 1. The properties of the decision units occurring in anticipatory networks

Decision unit	Type	Internal parameters	Output function(s)
Multicriteria optimizer	active	The feasible decision set U, the (vector) criterion F, the preference structure P	A single optimal solution or a subset of $\Pi(U,F,P)$
n-Player game unit	active	Strategy sets for all players, information sets, payoff functions	The values of payoff functions G_i for all players
Algorithmic decision unit	passive	The function D or a (deterministic) algorithm that calculates the value of the output function D on the set U	A unique value of D on $V\subset U$, or a subset of $D(U)$ determined by the inputs
Random Decision Unit	passive	Probability distributions describing the random decision generation	A random number or a random subset
Other units	active or passive	Different uncertainty model parameters (fuzzy, possibilistic, fuzzy-random variables etc.) and decision sets	Different types of outputs, depending on the specificity of the decision unit

The influence relations form the causal graph Γ, where the nodes are decision units O and the edges correspond to the general relation R defined as "*there exist an influence relation between two nodes*". Although each DU has one output, different influence relations may depend on this output, so any node can influence many nodes. We will assume that the graph $\Gamma=(O,R)$ is a directed acyclic graph (DAG). The paper [10] contains the discussion of the transitivity of R, and of potential causal graphs with loops that might correspond to so-called strong anticipatory systems [3].

As already mentioned in Sec.1, an anticipatory network is a multigraph, which contains additional anticipatory feedback relations. They can be defined as follows:

Definition 1. Suppose that Γ is a hybrid causal network consisting of optimizers, game units and other DUs shown in Fig.1. Assume that an active decision unit O_i in Γ precedes another one, O_j in the causal order R. Then the *anticipatory feedback relation* between O_j and O_i in Γ is a requirement f imposing certain condition(s) on the anticipated output from O_j when regarded as influenced by the decision made at O_i. ■

By Def. 1, the existence of an anticipatory feedback between O_j and O_i means that:

(i) the decision maker at O_i is able to anticipate the decisions to be made at O_j;
(ii) the results of this anticipation are to be taken into account when selecting the decision at O_i.

Throughout this paper we will assume that the requirement *f* in Def.1 specifies certain subset $\{V_{ij}\} \subset U_j$ that contains decisions available at O_j, that the decision-maker at O_j should consider when selecting the decision to satisfy the decision maker at O_i. Usually, this assumption means that reaching certain levels of criteria or payoff functions F_j on V_{ij}, is of special importance to the decision maker at O_i. Such levels can be defined as reference sets [7].

Now we can formulate the formal definition of the hybrid anticipatory network.

Definition 2. A *hybrid anticipatory network* is a hybrid causal network with at least two active decision units and with an additional anticipatory feedback relation. ■

In addition, we will assume that the initial node in an anticipatory network must always be an optimizer. Anticipatory networks with both types of relations as well as forecasts and scenarios regarding the future decision units and influence relation parameters form an information model, which can be applied to solve decision problems at the initial nodes or to model the decision-making processes.

An example of a hybrid anticipatory network is given in Fig.2.

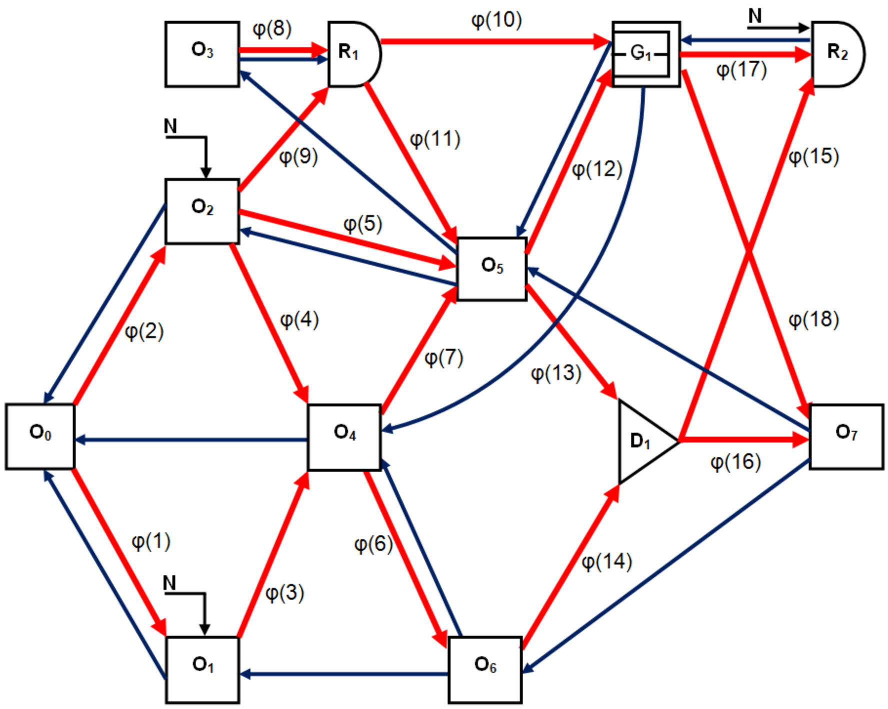

Fig. 2. An example of a hybrid anticipatory network: simple black boxes denote the optimizers, subdivided double boxes – the conflicting game decision units, triangles - the pre-determined algorithmic decisions, rounded elements – random decisions. Red arrows denote causal relations and temporal order; narrower blue lines – anticipatory information feedback

Let us observe that the anticipatory network shown in the above figure does not pre-determine neither its decision-theoretic interpretation nor the solution method. The latter depends on the fact whether the network is asynchronous, i.e. only the time order, and not its absolute values, are relevant. Additionally, the input aggregation functions at DUs should be specified. A more elaborated taxonomy of anticipatory networks is left to future research.

In the next section we will provide an outline description of the method of solving the anticipatory networks for the decision problems with finite sets of alternatives.

3 Solving Hybrid Anticipatory Networks

Below we present a short overview of the solution methodology proposed in [10] to solve the above-defined class of problems, assuming that

(i) The initial decision problem $O_0=(U,F,P)$ is a multicriteria decision making problem with a finite number of alternatives U,

(ii) The goal of the decision maker at the initial node O_0 is to consider all causal relations and anticipatory feedbacks in the network to select a single compromise decision or to confine the selection of a compromise decision to a possibly minimal subset of U,

(iii) In case where the satisfaction of all anticipatory feedback conditions yields no admissible solution, the decision maker accepts compromise solutions that satisfy certain relaxed condition (here we assume that the relaxation is based on a proximity measure to the subsets V_{ij}). Furthermore, the decision maker requires that the satisfaction of the anticipatory feedback conditions at the decision units in a closer future has a priority over feedback conditions at later moments,

(iv) The inputs to the same decision unit in the network manifest their influence simultaneously,

(v) For the clarity's sake, all decision unit parameters are assumed deterministic.

Definition 3. A hybrid anticipatory network is termed *solvable* if the process of considering all future information feedback results in selecting a non-empty solution set at the starting problem (or to each of the starting problems). ■

The general idea of algorithms used to solve anticipatory networks is be based on analysing cycles in the common network of causal and anticipatory feedback relations. Specifically, simple cycles, i.e. cycles, which do not contain other cycles, are replaced by a synthetic decision unit with a reduced set of admissible decisions and updated links to the remaining units in the network. The process is repeated until all cycles are solved and the network is reduced to a single simple cycle.

For decision problems with discrete sets of alternatives the analysis is performed on sequences of decisions at consecutive DUs ordered causally from the initial node to those having no successors. They are termed *admissible chains*, while those of them which fulfil all anticipatory feedback conditions are termed *anticipatory chains* (of decisions). Constructive solution algorithms for the networks of optimizers, based on dynamic programming have been given in [10]. Their construction assures that the above outlined procedure is convergent and yields a decision recommendation based on anticipated consequence scenarios.

The assumptions (i)-(v) above allow to formulate the following decision problem with relaxed anticipatory feedback requirements. Its solution for the optimizer networks is given in [10].

Anticipatory Decision-Making Problem (ADMP). For a hybrid anticipatory network Γ with finite decision sets find the set of all admissible chains $(u_1,\ldots,u_n)$ that maximize the function

$$g(u_0,\ldots,u_n):=\sum_{i\in J(0)}h(u_i,q(0,i))w_{0i} \tag{4}$$

and such that for all i, $1\leq i<n$, the truncated admissible chain $(u_i,\ldots,u_n)$ maximizes

$$g(u_i,\ldots,u_n):=\sum_{j\in J(i)}h(u_j,\ q(i,j))w_{ij}, \tag{5}$$

where $J(i)$, $i=0,1,\ldots,n$, denote the indices of decision units in Γ, which are in the anticipatory feedback relations with O_i, h is defined as

$$h(u_i,q(i,j)):=\left\| F_{i-1}(u_i)-q(i,j) \right\|, \tag{6}$$

and w_{ij} are positive coefficients corresponding to the relevance of each anticipatory feedback relation between the decision units O_i and O_j. ■

The decision making principle at O_0 implied by solving the above problem is to select this decision $u_0\in U$ that is the first element of an admissible chain satisfying (4)-(6). It turns out that taking into account anticipatory feedback conditions can lead to a considerable reduction of the number of compromise alternatives at the initial problem.

To solve the above presented discrete problem numerically, we have developed an application in Matlab, which consists of the following components:

1. A database W that contains all potential criteria, payoff functions, admissible alternatives for all decision stages U_i, $i=0,..k$, and all other components of decision units. It allows a data interchange with spreadsheets, definition of new units and a modification of those already stored in the database.
2. A graphical editor that makes possible an interactive construction of causal and anticipatory feedback networks. Each anticipatory network can be stored as a "problem file" that allows for its further processing and modifications.
3. A graphical module to define the multifunctions φ_i that describe the causal relations between a solution admitted and the scope of admissible decisions in some future problems. The same editor allows us to directly point out the elements of the sets V_{ij} that define the anticipatory feedback relations.
4. An analytic interface makes it possible to define all the graphs used in the problem solution in the form of lists of successors/predecessors, define the reference values q_i for the criteria or payoff functions that determine the sets $\{V_{ij}\}_{i\in I,j\in J}$, the functions $h(u_i,q_i)$ and coefficients w_{ij} that occur in problem (ADMP).
5. An analytic machine that implements dynamic programming algorithms given in [10] calculates the anticipatory chains, compromise solutions and their consequences in multicriteria anticipatory problems.

To conclude this section, let us observe that in the above presented approach to solving anticipatory networks we have assumed that the anticipation is a universal principle governing the solution of optimization problems at all stages. In particular, future decision makers modelled at the starting decision node O_0 can in the same way take into account the network of their relative future optimizers when making their decisions. Thus, the model of the future of the decision-maker at O_0 contains models of future agents including their respective future models. This has motivated us to introduce the notion of superanticipatory systems [8], that are direct generalizations of anticipatory systems in the sense of Rosen [5].

Definition 4. A *superanticipatory system* is an anticipatory system that contains at least one model of another future anticipatory system beyond itself. ■

In addition, one can introduce the notion of a grade of superanticipatory system, namely a superanticipatory system is of grade *K* if it contains the model of a superanticipatory system of grade *K-1*. Anticipatory systems, which are not superanticipatory are assigned grade *0*. One can observe that an anticipatory network containing a chain of *K* decision units, each one linked with the initial node O_0 and with all its causal predecessors in the same chain by an anticipatory feedback is an example of a superanticipatory system of grade *K*.

4 Discussion

This paper presents the fundamental ideas concerning hybrid anticipatory networks, the basic methods for solving them, and their extension, known as superanticipatory systems. From among a variety of anticipatory network architectures we have selected and analysed in more detail those that could be used to solve real-life problems in autonomous robot motion planning and to filter scenarios in technological foresight based on the identification of future decision-making processes and on anticipating their outcomes. Filtering plausible outcomes from each decision unit allows us to reduce the set of plausible action scenarios described as a chain of decisions and their implementation processes. This approach can also assist in planning the actions of cooperating autonomous robotic systems. Other applications include multi-stage resource allocation, predictive control of partitioned systems [11] and technological roadmapping.

Game-theoretic decision models that lead to mixed random strategies turned out to be a practical extension of anticipatory networks based on the multicriteria optimization principles presented in [10] for a situation where two non-cooperative future decision makers influence the same decision unit. *Scenarios* and *forecasts* can be used simultaneously to support decisions in hybrid anticipatory networks. Forecasts are an imminent component the model presented here, as the parameters of all future decision problems and the links between them need to be predicted. Scenarios can be external event-driven. When included in solution models, they allow us to generate decision rules that take into account the dependence of the next-stage problems-to-be-solved on the causally preceding problems as well as on potential outcomes of the external events considered.

The networking of future decision-making problems compelled us to introduce superanticipatory systems. The superanticipatory approach can change the paradigms of decision theory, where the decision is usually selected with a simple model of consequences. Our approach will require the decision maker to gather substantial information about anticipated future consequences, future decision problems and future decision makers. However, available information about the future has often been neglected or oversimplified due to the lack of appropriate models. The author believes that the theory presented in this paper and in [10] merits further research, such as an analysis of networks with iterative decision units, different inference models and multiple anticipatory feedback relations. The solution methodology for large anticipatory networks may require new, more efficient numerical methods as well, while evolutionary computational models seem particularly promising. Finally, detailed studies of real-life applications may provide clues as regards further directions of research on anticipatory networks and their applications.

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