Anticipatory Networks and Superanticipatory Systems

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Abstract: We present the theory of anticipatory networks that generalizes an earlier model of consequence anticipation in multicriteria optimization problem solving. We assume that the decision-maker takes into account the anticipated outcomes of future decision problem linked by the causal relations with the present one. So arises a multigraph of decision problems linked causally and by one or more anticipation relations termed here the anticipatory network. Then we will introduce the notion of superanticipatory systems, which are anticipatory systems that contain a future model of at least one more anticipatory system. Finally, we will present an application of anticipatory networks to filter the set of scenarios in a foresight project.

Keywords: anticipatory networks, superanticipatory systems, multicriteria optimizers, foresight

1. Introduction

This paper presents an introduction to the theory of anticipatory networks that generalizes the ideas related to anticipatory models of consequences in multicriteria optimization problems presented in [8] and [13]. We assume that when making a multicriteria decision, the decision-maker takes into account the anticipated outcomes of each future decision problem linked by the causal relations with the present one. In a network of linked decision problems the causal relations are defined between the time-ordered nodes. The future scenarios of the causal consequences of each decision are modelled by multiple edges starting from an appropriate node. The network is supplemented by one or more relations of anticipation, or anticipatory feedback, that describes a situation where decision-makers take into account the anticipated results of some future optimization problems while making their choice. Then they use the causal dependences of future constraints and preferences on the choice just made to influence future outcomes in such a way that they fulfill the conditions contained in the definition of the anticipatory feedback relations.

Both types of relations as well as forecasts and scenarios regarding the future model parameters form an information model, called here the *anticipatory network*. In Secs. 2 and 3 we will prove the basic properties of anticipatory networks and propose the method of their reduction, transformations and computing. We will also show that anticipatory networks with loops correspond to the strong anticipatory systems in the sense of Dubois [2], while the acyclic networks correspond to the weak ones.

In Sec. 3 we will present an application of anticipatory networks to select compromise solutions to multicriteria planning problems applying scenarios of anticipated consequences provided by an IT foresight project. Sec. 4 in we will analyze the anticipatory trees and general networks. Then, motivated by the properties of the anticipatory networks, we will introduce the notion of superanticipatory systems. By definition, a *superanticipatory system* is a system that is anticipatory in the sense of Rosen [7] or Dubois [2] (weak or strong) and contains a future model of at least one other anticipatory system which outcomes may influence its current decisions by an anticipatory feedback relation. This notion is idempotent, i.e. the inclusion of other superanticipatory systems into the model of the future does not yield an extended class of systems. We will observe that most anticipatory networks can be regarded as superanticipatory systems because future decisions can be based on similar anticipatory principles as the current one.

The motivation for the above outlined theory comes from a need to create an alternative approach to selecting a solution to multicriteria optimization problems that takes into account direct multi-stage models of the future consequences of the decision made which was presented in [8]. The anticipatory behavior of decision-makers correspond to the definition of anticipatory system proposed by Rosen [7] and developed further in a series of publications by Dubois and other researchers [2,1]. A bibliographic survey of these ideas can be found in [5]. An ability of creating a model of the future of the outer environment and of itself that characterizes an anticipatory system is also a prerequisite for an anticipatory network, where each node models an anticipatory system and they are able to influence each other according to the causal order. In this paper we restrict the anticipatory networks to model decisions made in networked optimization problems. Thus each decision node models an optimization problem forming thus a network of optimizers [13] − a new class of information processing systems introduced in [11]. Nevertheless, similarly to the anticipatory networks of optimizers, one can construct networks with nodes modeling Nash equilibria, set choice problems, random or irrational decision-makers, or hybrid networks containing nodes of all types. Some extensions of the theory of anticipatory networks are discussed in the final Sec. 5 of this paper.

2. Basic ideas of anticipatory networks

The original motivation idea behind introducing anticipatory networks as models of consequences was formulated in [8,11,13] as follows "*To use anticipated future consequences of a solution selected in a multicriteria decision problems as a source of additional preference information*". The exploration of anticipatory feedback in multicriteria decision making is possible owing to the following two assumptions:

- The decision maker responsible for solving an optimization problem included in the network knows how the parameters of future decision problems, with respect to this particular problem, are influenced by the solutions to preceding problems. This allows us to model the consequences of a decision to be made for any problem in the network.

- There exist estimates (forecasts or foresight scenarios) of future decision problem formulations, their solution rules, and the relations binding their anticipated outcomes with the current problem.

The anticipatory model of consequences presented in [8] has been extended in [11,13] to the theory of new networks, called *optimizers,* which model the optimization problems and their environment. According to a slightly more general definition given in [13], an *optimizer O* acts on a set of feasible decisions U and on the preference structure P and selects a subset $X\subset U$ according to P and to the fixed set of optimization criteria F that are characteristic for this optimizer. We will assume that the optimization problems solved by the optimizers have the form

$$(F:U\rightarrow E)\rightarrow min(P), \tag{1}$$

where P is a general preference structure in the sense of Yu and Leitmann [15] defined as $P:=\{\pi(u)\subset U: u\in U\}$ and such that if $v\in \pi(u)$ and $w\in \pi(v)$ then $w\in \pi(u)$. Usually E is a vector space with a partial order introduced by a convex cone θ, and

$$\pi(u):= \pi(u,\theta)=\{v\in U: F(v)\leq_\theta F(u)\}.$$

A *free optimizer O* may select any solution u from U that is nondominated with respect to P and F in (1), i.e. if

$$u\in \Pi(U,F,P):=\{u\in U: [\ \forall v\in U: F(v) \leq_\theta F(u)\Rightarrow v=u]\}.$$

O is then uniquely characterized by U, F, and P and may be denoted as a 3-tuple $O:=(U,F,P)$. If the admissible solution set in an optimizer O may be different from $\Pi(U,F,P)$ and equal to $X\subset \Pi(U,F,P)$, we will denote it as $O:=(U,F,P,X)$. X will be interpreted as the set of actually selected solutions to the optimization problem (1).

Besides of its optimizing capabilities, the optimizers may influence other optimizers, forming thus networks with some new properties compared to the theory of linked multicriteria problems. In particular, in feed-forward networks of optimizers constraints and preference structures in some optimizers are causally linked to the results of solving other problems and may depend on their preference structures. Thus, in a network of optimizers the parameters of the actual instances of optimization problems to be solved vary as results of solving other problems in the network.

An influence relation r represented by the network of optimizers may be defined as

$$O_1 :=(U_1,F_1,P_1,)\ r\ O_2 :=(U_2,F_2,P_2) \Leftrightarrow \exists\varphi:X_1 \rightarrow 2^{U_2}: X_2=\varphi(X_1).$$

Influence relations linking preference structures may be defined analogously. If r is acyclic it will be termed a *causal relation*. From this point on the term *causal network* will refer to the graph of a causal relation. In a causal network of optimizers the functions φ influence the constraints in O_i by outputs from the problems preceding O_i.

In [13] the anticipatory information feedback in causal networks of optimizers has been applied to selecting a solution to the optimization problem modeled by the starting element in the anticipatory optimizer network. Specifically, while making the decision, the decision maker takes into account the forecasts concerning the parameters of future decision problems, the anticipation concerning the behaviour of future decision makers, the forecasted causal dependence relations linking the parameters of optimizers in the network and the anticipatory relations pointing out relevant future outcomes to particular decisions to be made at nodes preceding them in the causal order.

Here, we will focus our attention on the problem of finding feasible foresight scenarios based on the identification of future decision-making processes and on anticipating their outcomes. Scenarios, such as those defined and used in foresight and strategic planning [3], can depend on the choice of a decision in one of the networked optimization problems as well as be external-event driven. When included in a causal network of optimizers, the anticipation of future decisions and alternative external events would allow us to generate alternative structures of optimizers in the network. Assuming that at each optimizer in the causal network the decision makers strive to select their decisions in a rational way, and applying multicriteria optimization methods to find all potential variants of anticipated future problem outcomes, one can find the set of potential elementary scenarios [12] of future trends and events modelled by the network. Considering additionally the anticipatory feedbacks in the network that are defined below, it makes possible a filtering of plausible outcomes from each problem, and the reduction of the set of plausible elementary scenarios. This application can be very assistive when building foresight scenarios by clustering elementary scenarios.

To complete the definition of anticipatory networks, we need first to define the anticipatory feedback relation.

Definition 1. Suppose that G is a causal network consisting of optimizers and that an optimizer O_j in G precedes another one, O_i, in the causal order r. Then the anticipatory feedback between O_j and O_i in G is information flow concerning the anticipated output from O_i to be regarded as an input to the optimizer O_j. ■

By the above definition, the existence of an information feedback between the optimizers O_n and O_m means that the decision maker at O_m is able to anticipate the decisions to be made at O_n and that the results of this anticipation are to be taken into account when selecting the decision at O_m. This relation does not need to be transitive. As in the case of causal relations, there may also exist multiple types of anticipatory information feedback in a network, each one related to the different way the anticipated future optimization results are considered at a decision node O_m. This information is then used to specify the selection rule of the optimizer O_j so that the results of the constrained optimization at O_j were optimal in the sense of optimization with variable constraints [9]. Both relations, the causal influence described by the network G, and the anticipatory feedback, when considered jointly and expressed in a diagram, form an anticipatory network of optimizers:

Definition 2. *An **anticipatory network** (of optimizers) is a causal network of optimizers with at least one additional anticipatory feedback relation.* ■

Now we will present a few key definitions referring to the anticipatory networks of optimizers.

Definition 3. *Any two optimizers $O_m=X_1(U,F,P)$ and $O_n=X_2(W,G,R)$ are in the causal influence relation if there exist two different outputs from O_m, $x_1,x_2 \in X_1$, such that either the choice in O_n is restricted to two different subsets of W that depend on choice of x_1 or x_2 in O_m, or if the solution selection rule R or the criterion G are modified in different manner depending on the choice of x_1 or x_2.*

An example of an optimizer network with information feedback is given in Fig.1.

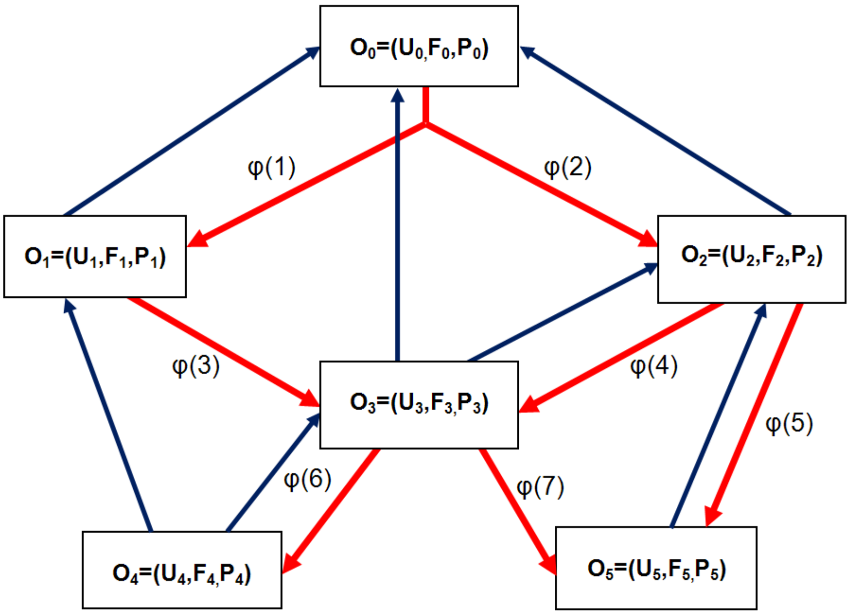

Fig.1. An example of a network of optimizers: solid red lines denote causal relations and temporal order; narrower blue lines denote anticipatory information feedback.

In the network diagram presented above, the solid red lines denote *direct causal influences*. When studying the relations between the optimization problems that model future consequences of a decision made it is convenient to consider the weak (or potential) causal dependence relation, which is the complement of the relation "*O_n cannot be influenced by O_m*". Observe that an optimizer O_m is potentially influenced by O_n if in the influence network they are connected by a path of direct causal influences leading from O_n to O_m, called here a *causal path*. It is easy to see that the existence of a causal path between O_n and O_m does not guarantee that the Def. 4 is fulfilled. When building an anticipatory network it is convenient to assume that the *weak causal* *influence relation* so defined is transitive taking into consideration the transitive extension of the union of actually observed partial causal influences and the complement of influence exclusions. On the other hand, the causal influence relation (cf. Def.4) may be defined in such a manner that O_j influences O_m and O_m influences O_n, but the choice of solutions of O_j has no influence on the outputs from O_n.

Let us observe that the causal component of an optimizer network can be represented as the Hasse diagram of a weak causal influence relation as shown in Fig.3. Since in a real life situation the nature of future causal influences is uncertain, it is at the same time the Hasse diagram of an unknown causal influence relation (cf. Def.3) that need not be transitive, contains no more elements than the weak causal relation, but at least as much as its transitive reduction. Specifically, if two optimizers O_m and O_n are connected by a causal path, it means that there potentially exists a causal influence, consequently, the information feedback between them makes sense. On the contrary, a lack of such path means that seeking an information feedback between O_n and O_m will fail. In general, for given optimizers O_k, O_n and O_m, there may exist different ways of influencing O_m by O_n, both direct and indirect, the latter being superpositions of direct relations. Therefore, in general case, a causal diagram of an anticipatory network could be a multigraph.

When analyzing networks of optimizers with anticipatory information feedback relations it is sufficient to take into account only *essential information feedbacks (EIF),* by definition O_n and O_m are linked by an EIF relation iff there exist both an information feedback between O_n and O_m and a weak causal influence between O_m and O_n. Indeed, if there is no possibility that a decision made at O_m can influence the decision choice at O_n then any forecast of the future decision maker's behavior at O_n is useless. This is discussed further in the next sections, where we will also solve the anticipatory networks, according to the following definition.

Definition 4. *An* ***anticipatory network*** *(of optimizers) is said to be solvable if the process of considering all anticipatory information feedbacks results in selecting a non-empty solution set at the starting problem* ■

To analyse networked optimizers, we will admit several assumptions:

(1) The solution to an optimization problem O_p may directly influence a finite number m_p of subsequent problems;
(2) An optimization problem O_p may be directly influenced by a finite number n_p of preceding problems;
(3) The aggregation rules for different influencing factors are defined for each optimizer influenced by more than one predecessor (e.g. as intersection of the sets of feasible alternatives, each one imposed by a different preceding optimizer);
(4) An optimization problem O_p may be linked by a finite number j_p of information feedbacks with anticipated solutions of influenced future problems.

The general underlying idea behind the procedures proposed in this paper is to analyze chains of optimizers linked by a causal influence relation, then to identify in a network of optimizers elementary cycles consisting of causal influence along chains and future information feedback relations, i.e. cycles, which do not contain other cycles. For chains of optimizers we proposed a numerical solution procedure (Algs. 1 and 2 in [13]) based on an analysis of above-defined elementary cycles, starting from those most distant in time, and on replacing a solved elementary cycle by a synthetic optimizer and updated links to the remaining elements of the network, the latter as the links of the outer elements of the elementary cycle just solved. The process is repeated iteratively until all cycles are solved. A general network can be decomposed into chains, which makes it possible to apply aggregated chain rules iteratively, gradually eliminating solved chains.

3. Chains of optimizers

We will study anticipatory networks of optimizers with the causal influence relation defined by linking multifunctions

$$Y_i{:}F_{i-1}(U_{i-1})\rightarrow\rightarrow U_i,\ \varphi(i){:=}Y_i \circ F_{i-1} \tag{2}$$

imposing additional constraints in sets U_i where the notation "$a{:}A\rightarrow\rightarrow B$"is used for $a{:}A\rightarrow 2^B$. The dependence of preference relations P_i on the outcomes of previous problems is defined by the functions

$$\psi{:}\ X(U_{i-1},F_{i-1},P_{i-1})\ni f\rightarrow P_i.$$

A simple causal graph of optimizers that can be embedded in a straight line will be called a *chain* of optimizers. The Fig.2 contains an example of a chain of optimizers, where a decision maker at the future optimizer $O_1=(U_1,F_1,P_1)$ will make the decision taking into account the anticipated outcomes at O_2 and O_4, while the decision maker at the initial problem O_0 makes the decision based on the model of O_1, which is itself an anticipatory system, and on the outcomes of other optimizers.

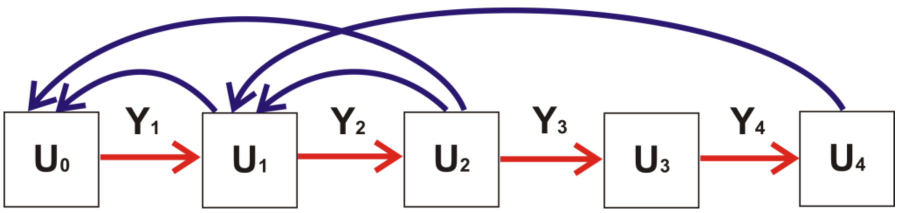

Fig. 2. An example of the symbolic representation of a chain of optimizers with anticipatory feedback consisting of five elements $O_i=(U_i,F_i,P_i)$, $i=0,1,\ldots 4$, with $F_i{:=}id_{U_i}$, therefore $\varphi(i)\equiv Y_i$, U_i ordered by P_i, causal relations defined by multifunctions Y_i, and four anticipatory feedback relations (blue arrows). The temporal order is determined by causal relations (solid red arrows).

Chains of optimizers are the simplest, yet important, class of optimizer networks, since a large class of such networks can be reduced to a subsequent analysis of all chains in a network. It can be observed that the evolution of compromise selection mechanisms ψ_i in a chain of optimizers can be modelled in a similar way as the influence on constraints and included in the same model as an additional type of causal relations.

An theoretical base for solving the initial problem $O_0:=(U_0,F_0,P_0)$ in a chain of optimizers O_i $i=0,1,\ldots,N$, with discrete sets U_i, is given below. Causal relations between optimizers are given in the form of restrictions in the scope of admissible decisions defined as multifunctions $\varphi(i)$ that depend on solutions of previous problems modelled by O_{i-1}, for $i=1,\ldots,N$. Future information feedback is defined as information about anticipated fulfilment (or not) of certain conditions by the values of criteria in future optimization problems. The following definitions will be helpful to describe the solution procedure in a more rigid manner.

Definition 5. *Let O_i $i=0,1,\ldots,N$, be a chain of optimizers. For each $i=0,\ldots,N-1$ let $J(i)\subset\{i+1,\ldots,N\}$ be the set of future problems – possibly empty – of which anticipated outcomes are considered when making a decision at the i-th stage; equivalently, there exist information feedbacks between the i-th node and all m-th nodes, when $m\in J(i)$. The sets $J(i)$ will be called* ***feedback indices*** *for the i-th optimizer.* ■

Remark: The theory remains valid if we allow self-feedback loops, i.e. if $J(i)\subset\{i,\ldots,N\}$ that describe interactive decision-making at the same optimizer. However, for the brevity of presentation, we will not consider such problems in this paper.

Definition 6. *Let O_i $i=0,1,\ldots,N$, be a chain of optimizers with causal relations given by the restrictions in the scope of admissible decisions in U_{i+1} such that if u_i is an admissible decision for the optimizer O_i then u_{i+1} is an admissible decision in U_{i+1} iff $u_{i+1}\in\varphi(i)(u_i)$, where $\varphi(i):=Y_i \circ F_{i-1}$ is a multifunction describing the restrictions in the admissible choice of solutions in optimization problems subsequent to O_i. Moreover, let us assume that all elements of U_0 are admissible. Then any sequence of admissible solutions $(u_{0,m(0)}, u_{1,m(1)},\ldots, u_{N,m(N)})$ will be called an* ***admissible chain***. ■

The set of admissible chains will be denoted by A. If all sets $U_0,\ldots,U_N$ are finite then A can be constructively generated by listing subsequently the elements of U_0, then, in the next step, replicating each row of the list corresponding to u_{0i} c_{1i}-times, where c_{1i} is the cardinality of $\varphi(1)(u_{0i})$ and adding to each row of the extended list the values of $\varphi(1)(u_{0i})$,. Then one can proceed recursively, replicating at the *i*-th step each row corresponding to the *j*-th element of U_i c_{ij}-times, where c_{ij} is the cardinality of $\varphi(i)(u_{i-1,j})$, until the elements of U_N are added to the list. An example of the digraph that corresponds to the above procedure is shown in Fig.4.

By virtue of the following Lemma the enumeration of the set A can be accomplished without listing all admissible chains.

Lemma 1. *Let us denote by M the number of all admissible chains in a chain of optimizers $O_0,\ldots,O_N$ with finite $U_0:=\{u_{0,1},\ldots,u_{0,k(0)}\},\ldots,U_N:=\{u_{N,1},\ldots,u_{N,k(N)}\}$ and by d_{ij} the number of partial admissible chains starting at U_0 and ending at $u_{ij}\in U_i$ for $i=1,\ldots,N$; $j=1,\ldots,k(i)$. Then*

$$d_{ij}=\Sigma_{1\leq p\leq b_{ij}}\, d_{i-1,p},$$

where b_{ij} is the cardinality of the set $\{u_{i-1,p}\in U_{i-1}: u_{ij}\in\varphi(i)(u_{i-1,p})\}:=\varphi(i)^{-1}(u_{ij})$. Consequently, $M=\Sigma_{1\leq p\leq k(N)}\, d_{Np}$.

Proof: *Let us observe that finding all admissible chains A is, by definition of A, equivalent to finding all paths in a stratified digraph G(A), where nodes correspond to the elements of $U_0,\ldots,U_N$ and the edges can link the elements of U_{i-1} and U_i only, for $i=1,\ldots,N$. By definition, an edge $(u_{i-1,j},u_{ip})\in G(A)$ iff $u_{ip}\in\varphi(i)(u_{i-1,j})$. Observe, moreover, that for each $u_{1p}\in U_1$ $d_{1p}=b_{1p}$. Then by induction and from the construction of G(A) we conclude that $d_{ij}=\Sigma_{1\leq p\leq b_{ij}}\, d_{i-1,p}$. Summing up the cardinalities of the sets of all paths ending at different elements of U_N yields the latter formula.* ■

The value of *M* in Lemma 1 is an estimate of the maximum number of paths that would have to be surveyed when analyzing a chain of optimizers. From the above Lemma it follows immediately that in the worst case when the multifunctions $\varphi(i)$ impose no additional constraints on the decisions at $O_1,\ldots,O_N$, i.e. if $\varphi(i)(u_{i-1,j})=U_i$ for $i=1,\ldots,N$, $u_{i-1,j}\in U_{i-1}$, then the number of all admissible chains *M* is equal to $\Pi_{0\leq i\leq N}\, k(i)$. However, in such a situation there would be no causal dependence between decisions at different optimizers and no essential information feedbacks could be defined. In the second extreme case when $\varphi(i)(u_{i-1,j})$ always contains one element, i.e. if future decisions are functionally determined, *M* is equal to *k(0)*.

The set of admissible chains *A* will be filtered by imposing additional conditions, related to the anticipated future optimization outcomes.

Definition 7. *For a chain of optimizers O_k $k=0,1,\ldots,N$, let us define the* ***anticipatory feedback*** *condition at O_i, $0\leq i<N$, as the requirement that*

$$\forall j\in J(i) \text{ for a given family of subsets } \{V_{ij}\}_{j\in J(i)}\ u_j\in V_{ij}\subset U_j. \qquad (3)$$

For all O_i such that the set *J(i)* is non-empty, the feedback can be described in the following way: the decision maker at O_i strives to select the solution which guarantees that the anticipatory feedback condition (3) is satisfied for all $j\in J(i)$. The sets V_{ij} represent the recommended or desired decisions to be made at the *j*-th optimizer from the point of view of the decision maker that is responsible for the outcomes from the *i*-th optimizer. Usually, it means that the criteria values on V_{ij}, $F_j(V_{ij})$ are of special importance to the decision makers and can be defined as reference sets [10] but in the discrete problems considered in this paper they may be identified with the elements of V_{ij}.

We will also assume that the sets *J(i)* and all V_{ij} are non-empty for at least one *i*, and if $J(i)\neq\emptyset$ then $V_{ik}\neq U_k$ for at least one $k\in J(i)$. Otherwise the chain of optimizers would be called *trivial*. If a causally final optimizer O_N (i.e. no optimizer O_k depends on O_N) is a starting node for an anticipatory feedback then the chain will be called *non-redundant*, otherwise it will be called *redundant*. In the latter case, the causal relations beyond last starting node for an anticipatory feedback have no effect on calculating the chains fulfilling anticipatory feedback condition (3) and can be omitted.

In a non-trivial chain of optimizers the following decision problem can be formulated:

Problem 1. *Find the set of all admissible chains that additionally fulfil anticipatory feedback condition (3).*

However, when modelling real-life problems it may turn out that the inclusions generated by the sets V_{ij}, are likely to be too restrictive, so that no feasible solutions to the Problem 1 can be found. Therefore we will relax the condition (3) by allowing its partial fulfilment, at some $j\in J(i)$ only. The following Problem 2 is an example of such relaxation. Without a loss of generality we will assume that $J(0)\neq\emptyset$.

Problem 2. *Find the set of all admissible chains* $(u_1,\ldots,u_N)$ *that minimize the following function:*

$$g(u_0,\ldots,u_N):=\sum_{i\in J(0)}h(u_i,q(0,i))w_{0i} \tag{4}$$

and such that for all i such that $J(i)\neq\emptyset$, $1\leq i<N$, *the truncated admissible chain* $(u_i,\ldots,u_N)$ *minimizes the function*

$$g(u_i,\ldots,u_N):=\sum_{j\in J(i)}h(u_j,q(i,j))w_{ij}, \tag{5}$$

where h is certain quantitative measure of satisfaction of (3), e.g.

$$h(u_i,q(i,j)):=\min\{\|F_i(u_i)-y\|: y\in F_i(U_i) \text{ and } y\leq q(i,j)\} \tag{6}$$

and w_{ij} *are positive coefficients corresponding to the relevance of each anticipatory feedback relation between the optimizers* O_i *and* O_j.

The key notion in this paper can now be defined as follows:

Definition 8. *The solutions to either Problem 1 or 2, a sequence of decisions* $u_{0,m(0)},\ldots,u_{N,m(N)}$, *fulfilling (3) or minimizing (4)-(5), will be called* ***anticipatory chains*** *of type 1 or 2, respectively.* ■

Based on the recursive definition of the solutions to Problems 1 or 2, it is easy to see that the following observation is true:

Proposition 1. *Suppose that* $\{u_{k,m(k)},\ldots,u_{N,m(N)}\}$ *is an anticipatory chain for the optimizer* O_k *in a non-redundant chain of optimizers. If J(n) is non-empty for a certain* $n\in[k+1:N]$, *then* $\{u_{n,m(n)},\ldots,u_{N,m(N)}\}$ *is an anticipatory chain for the optimizer* O_n.

Proof: The above optimality principle follows from the observation that if $(u_{0,m(0)},\ldots,u_{N,m(N)})$ is an anticipatory chain then from the construction of the anticipatory network it follows that for each $n\in[1:N]$ such that $J(n)\neq\emptyset$ the solution $u_n\in U_n$ has been determined so that the chain $\{u_{n,m(n)},\ldots,u_{N,m(N)}\}$ is anticipatory. The chain of optimizers is finite, therefore there exists the largest integer M such that $J(M)$ contains N, specifically $N\in J(M)$ because we assumed that the chain is non-redundant. After $u_{M,m(M)}$ and $h(u_{M,m(M)},q(M,m(M)))$ are determined explicitly, the backward calculation process yields an anticipatory chain by induction. ■

The above Proposition 1 allows us to derive a constructive computational procedures (cf. [13) based on the dynamic programming principle to find

a) the best anticipatory non-dominated solution to (1) and the corresponding anticipatory chain for the chain of optimizers with finite sets U_i
b) all anticipatory chains in an anticipatory network.

4. Anticipatory Trees and General Networks

In this section we will demonstrate that the solution approach presented in Sec. 3 for anticipatory chains can easily be generalized for the case where each optimizer can influence multiple decisions to be made in the future and that do not depend on each other. If such a situation occurs, it can be represented as a *causal tree* of optimizers. Then we will investigate a situation where each future decision may depend on the outcomes of multiple mutually-independent decisions. Both cases combined can be represented as *causal networks* with anticipatory feedbacks.

Let us first formulate the following two definitions:

Definition 9. *Let the optimizer O_i influence causally the decisions in two mutually causally-independent optimizers O_k and O_m and let O_t be an optimizer with the following properties:*

(a) O_t is causally dependent on O_i,

(b) O_k and O_m are both causally dependent on O_t,

(c) If O_p is causally dependent on O_t then O_k and O_m cannot both causally depend on O_p.

*Then O_t will be called a **bifurcation optimizer** for O_i, O_k and O_m.* ■

Definition 10. *An **anticipatory tree** is a finite network of optimizers that contains at least one bifurcation optimizer, at least one anticipatory feedback, and such that no optimizer in the network depends on two or more causally-independent optimizers.* ■

By an *end node* of an anticipatory tree T we will mean any optimizer in T that does not causally influence any other optimizer. The *final branch* of T is defined as the chain that starts from a bifurcation optimizer and does not contain any other bifurcation optimizers.

In general, it can be observed that for given O_i, O_k and O_m such as in Def.6, the bifurcation optimizer O_t always exists, but needs not be unique. However, from Def.7 it follows that it is always unique in an anticipatory tree of optimizers. An example of a bifurcation optimizer in a simple tree is given in Fig. 5.

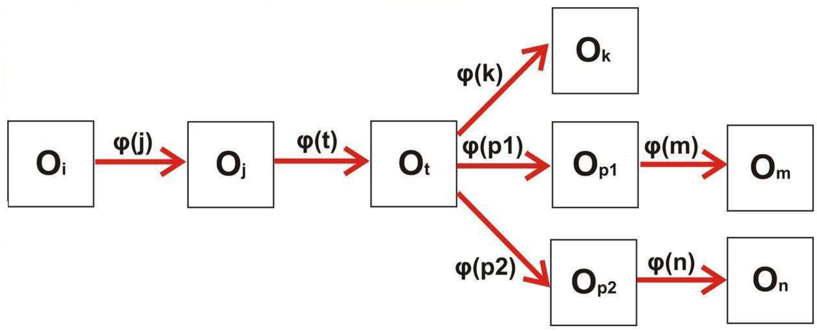

Fig. 3. An example of a simple tree of optimizers, where O_t is the bifurcation optimizer for O_m, O_n. O_k, O_{p1} and O_{p2}. Causal relations are defined by the multifunctions $\varphi(i):=Y_i \circ F_i$. Anticipatory feedback relations are not shown in this figure.

An example of an anticipatory network with two bifurcation optimizers for O_i, O_{k1} and O_{k2} is shown in Fig.4.

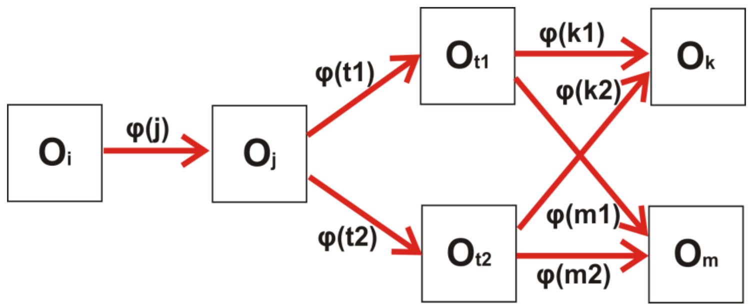

Fig.4. A network of optimizers, where both O_{t1} and O_{t2} are bifurcation optimizers for O_{k1} and O_{k2}

The following property makes possible the reduction of the analysis of anticipatory trees to the subsequent analysis of anticipatory chains in the tree.

Proposition 2. *Assume that the optimizer O_i influences two causally independent optimizers O_k and O_m in an anticipatory tree T and let O_t be the (unique) bifurcation optimizer for $O_{i,}$,O_k and O_m. Furthermore, let C_k and C_m be the sets of admissible chains starting at O_i and ending at O_k and O_m, respectively, and let C_k' and C_m' contain the elements of C_k and C_m, respectively, starting at O_i and truncated at the bifurcation optimizer O_t. Then the set of all admissible chains with respect to both O_k and O_m, starting at O_i can be generated as extensions of the elements of the intersection of C_k'*

and $C_m{}'$. More precisely, an extension of any such sequence of decisions starting at O_i and ending at O_t is to be concatenated with an arbitrary subsequence starting at O_t of an admissible chain that was truncated at prior to the intersection of $C_k{}'$ and $C_m{}'$.

Proof: Without a loss of generality we can assume that in the anticipatory tree T there is only one element O_0 such that O_0 does not depend on any other optimizer. Actually, if there are two such elements, $O_0{}'$ and $O_0{}''$ then by Def.7, no element of the tree T can depend on both $O_0{}'$ and $O_0{}''$ so that in such a case the tree T would consist of two disjoint parts T' and T'' with $O_0{}' \in T'$ and $O_0{}'' \in T''$ Observe now that the anticipatory tree T can be represented in a unique way as a union of maximum anticipatory chains, i.e. such that each one starts at O_0 and ends at a *final branch*, i.e. an optimizer that does not have any causal successors. Such decomposition exists because we assumed that the tree T is finite (Def.7). Consequently, there is a finite number of final branches, and each one defines an anticipatory chain between O_0 and this final branch, which is unique because the causal graphs contain no loops. Then the construction provided in Prop.2 follows directly from the definitions of the causal dependence and the set of admissible chains A (Def.3). ■

Corollary. *Let A_1 and A_2 be the sets of admissible chains in two chains of optimizers $\{O_i,\ldots,O_j,\ldots,O_k\}$ and $\{O_i,\ldots,O_j,\ldots,O_m\}$, respectively, where O_j is their bifurcation optimizer. Only those elements of A_1 and A_2 that overlap on the common branch $\{O_i,\ldots,O_j\}$ of the anticipatory tree $T:= \{O_i,\ldots,O_k\}\cup\{O_i,\ldots,O_m\}$ can be prolonged to chains admissible with respect to causally independent consequences represented by the decisions made at O_k and O_m.* ■

Observe that if there exist anticipatory feedbacks between causally independent O_k and O_m, and their common causal predecessor O_i, then a similar construction to that presented in Prop.2, based on the logical product of feedback conditions and the resulting intersection of chains, will also apply to the anticipatory chains. On the contrary, anticipatory feedback between causally independent O_k and O_m, is always irrelevant, i.e. by definition there is no action at O_k that might influence the output of O_m. Therefore from Prop.2 and the above observations one can derive a constructive computational procedure for solving anticipatory trees [13].

In Fig. 5 we will show a more complicated tree T that consists of ten optimizers in four chains. This will serve as an example of the operation of such a procedure.

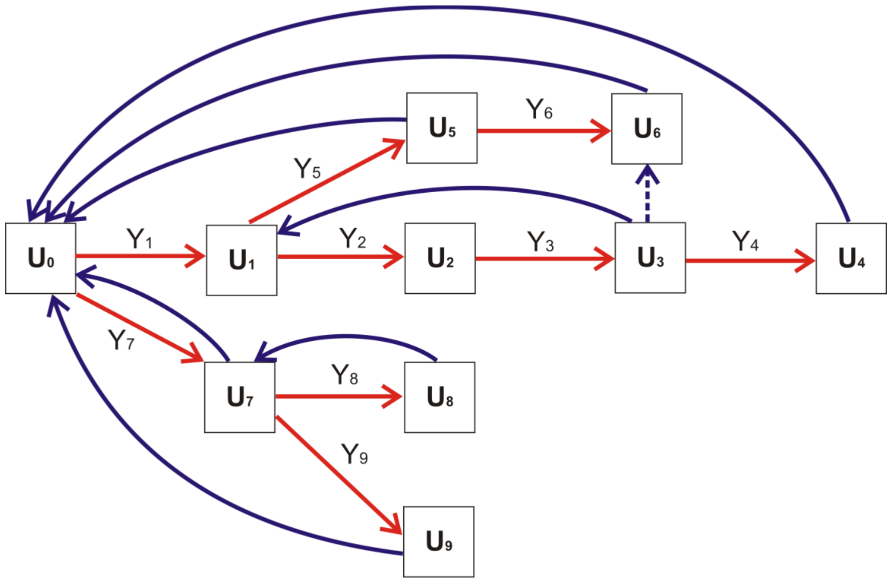

Fig. 5. An example of a symbolic representation of a tree T of optimizers with anticipatory feedback consisting of ten elements $O_i=(U_i,F_i,P_i)$, $i=0,1,\ldots 9$, and four chains, with $F_i:=id_{Ui}$, causal relations defined by multifunctions Y_i, and seven essential anticipatory feedback relations (solid arrows). The dotted arrow between U_3 and U_9 is a non-essential anticipatory feedback, because there is no causal relation between these optimizers. The temporal order within each chain is determined by causal relations, there exist optimizers with no causal relationship.

First, all anticipatory feedback relations are checked whether they link weakly causally dependent optimizers in an appropriate order. For instance, if the anticipatory feedback links O_i and O_j, this can be accomplished by an iterative search in the list of causal predecessors of O_i until either O_j or O_0 is reached. In the latter case the feedback is non-essential. In the example shown in Fig. 5 the feedback between O_3 and O_6 is identified as non-essential and eliminated from further consideration. There are four end nodes in the tree, namely O_4, O_6, O_8, and O_9. According to Def. 9, O_1 is the bifurcation optimizer for the end nodes O_4 and O_6 and O_7 is the bifurcation optimizer for O_8 and O_9, so after surveying all end nodes $B=\{O_1,O_7\}$. Pursuing the procedure, we remove the final branches of T which correspond to the bifurcation optimizers identified so far, or – equivalently - all optimizers causally dependent on either O_1 or O_7. The only bifurcation optimizer for the reduced tree is O_0 which is now added to B. Thus B consists of three bifurcation optimizers: O_1,and O_7 and O_0.

Generally, in a network of optimizers there may exist units that are influenced causally by two or more predecessors (cf. Fig.1). Such problems may emerge in practice when, for example, an input to a production function comes from two independent technological processes, which are both optimized with respect to quality and price. In order to deal with such a situation, observe first that the causal dependences in form of constraints on the set of admissible decisions in a subsequent problem O_k that comes from two or more causally independent optimizers $O_i=(U_i,F_i,P_i)$ and $O_j=(U_j,F_j,P_j)$ as the multifunctions Y_i and Y_j, *respectively*, yield, in fact, just an intersection of constraints that can be represented as a new multifunction Y defined on the Cartesian product of

F(U_i) and F(U_j) in the following way

$$Y(u_{ip},u_{jr}) := Y_i(u_{ip}) \cap Y_j(u_{jr}).$$

Based on this observation, in case of arbitrary networks, the calculations can again be reduced to an analysis of chains, and elementary loops in the network, i.e. loops which may consist of both, causal relations and anticipatory feedbacks, and do not contain other loops. Analogously to surveying the bifurcation optimizers and 'cutting the branches' of an anticipatory tree, one has to survey all optimizers which are causally influenced by two or more predecessors. If an elementary loop is detected, it can be replaced by a synthetic optimizer with a reduced set of admissible chains and updated links to the remaining elements of the network. The process can be repeated iteratively until the 0^{th} optimizer has been reached. The procedure of finding anticipatory chains, referring to Problem 1, can be regularised for a case where no solution exists, by solving the relaxed Problem 2 in the manner presented in Sec.3.

To conclude this section, let us observe that in the above presented approach to solving anticipatory networks we have assumed that the anticipation is a universal principle governing the solution of optimization problems at all stages. In particular, future decision makers modelled at the starting decision node O_0 can in the same way take into account the network of their relative future optimizers when making their decisions. Thus, the model of the future of the decision-maker at O_0 contains models of future agents including their respective future models. This has motivated us to introduce the notion of superanticipatory systems, that are direct generalizations of anticipatory systems in the sense of Rosen [7] and Dubois [2]:

Definition 11. *A superanticipatory system is an anticipatory system that contains at least one model of another future anticipatory system.* ■

Since, by definition, every superanticipatory system is also anticipatory, the class of superanticipatory systems remains closed when an anticipatory system contains a model of a superanticipatory one. However, one can introduce the notion of a grade of super-anticipatory system, namely a superanticipatory system is of grade n if it contains the model of a superanticipatory system of grade *n-1* and defining anticipatory systems and supeanticipatory of grade *0*. One can observe that an anticipatory network containing a chain on n optimizers, each one linked with O_0 and with all its causal predecessors by an anticipatory feedback is an example of a superanticipatory system of grade *n*.

5. The computational aspects of anticipatory networks

To solve the problems presented in Secs. 3 and 4 one requires information about the future optimization problems and their mutual relations. If the time horizon of anticipatory planning is large compared to the time allotted to modelling and computing the decision, usually also the changes in the modelled environment proceed slowly enough that allows an analyst to rely on the information gathered prior to performing all computations. This is the case of foresight applications, where the time horizon is usually between 10 and 20 years, the analytic phase can be stretched over several months and the resources available allow us to explore the future to a sufficient extent. For this class of applications an implementation of Algs. 1 and 2 in an off-line computational environment such as Matlab turned out satisfactory. A prototype application in Matlab consists of the following components:

1. A database *W* that contains all potential criteria, admissible alternatives for all decision stages U_i, $i=1,..k$, and the optimizers, pre-defined or constructed from basic elements (criteria, alternatives). The preference structures can be defined and associated with optimizers as one of the management functions of the database. It allows a data interchange with spreadsheets, definition of new optimizers and a modification of those already stored in the database.
2. A graphical editor that makes possible an interactive construction of causal and anticipatory feedback networks. It operates on a symbolic graphical representation of optimizers that have been defined previously. Each network is stored as a "problem file" that can be further processed and modified.
3. A graphical module to define the multifunctions Y_i that describe the causal relations between a solution admitted and the scope of admissible decisions in some future problems. The same editor allows us to directly point out the elements of the sets $V_{i,j}$ that define the anticipatory feedback relations.
4. An analytic interface makes it possible to define all the graphs used in the problem solution in the form of lists of successors/predecessors, define the reference values q_i for the optimization criteria that determine the sets $\{V_{i,j}\}_{i\in I, j\in J}$, the functions $h(u_i,q_i)$ and coefficients $w_{i,j}$ that occur in Algorithm 1. It is also possible to assign time intervals to optimizers, which can be useful when analyzing problems arising in a foresight context.
5. An analytic machine that implements Algorithms 1 and 2 (given in the Appendix), calculates the anticipatory chains, compromise solutions and their consequences in multicriteria anticipatory problems, and visualizes the results.

The application makes it possible to manage different problems analyzed previously or simultaneously. The results obtained for the same initial optimizer but using different causal and future information structures can be compared with each other. We conclude that a similar solution scheme to that presented in Alg. 1 and 2 can be used as a base to implement a multicriteria problem solution environment for general anticipatory networks. that – as estimated from computational evidence - allow us to efficiently manage problems with up to 10^3 alternatives in up to 10^2 optimizers. The analysis of larger or more specialized problems will require tailored decision support systems capable of using anticipatory information.

6. Final remarks

This paper presented the basic ideas concerning the anticipatory networks, the basic methods to solve them, and their extension, termed superanticipatory systems. Anticipatory networks may be applied to model and solve a broad range of problems, both real-life and theoretical. Apart from the above-mentioned inspirations coming from potential uses in foresight, roadmapping, finance, and environmental modelling, there are further potential fields of application, such as

- Anticipatory analysis of queuing networks;
- Policy planning: an opportunity to apply the quantitative results of foresight, scenarios, and impact analysis in a uniform model;
- Extending the theory of multi-step Stackelberg games, *n*-level programming;
- Financial and real investment support systems, including multi-step project portfolio management
- Adaptive robot control systems with feedback.

They can also contribute to model foresight processes in a clear, formal way.

Anticipatory networks can be regarded as a new class of world models where the present and future decisions are described as the results of solving optimization problems and are mutually linked by causal relations. The importance of using world models to elicit and assess the consequences of choice in causal decision making has been acknowledged by some authors (cf. e.g. [4,6,14]), but these have been still described as expected utilities. In this paper we have emphasized that the consequences of a decision made should be considered in a complex environment that needs heterogeneous modeling techniques. Anticipation based on the knowledge of decision-making principles to be applied at future problems together with forecasts and scenarios concerning the parameters of future processes allows the decision-maker to choose a compromise decision that creates the 'best possible' future in terms of consequences taken into account in the anticipatory network.

The anticipation model presented in this paper is nested, which means that any decision agent included in the model of future consequences of a decision to be made does also possess the ability to anticipate the results of its decisions and that the behavior of such agents is a component of our model. This was the idea that motivated us in a natural way to introduce the superanticipatory systems. The superanticipatory approach can change the paradigms of multicriteria decision making, where the compromise decision has been usually selected with a simple model of consequences expressed by a value or utility function. Our approach will require the decision maker to gather much information about anticipated future consequences, future decision problems and future decision makers. However, available information about the future that has been previously neglected or simplified excessively due to the lack of appropriate models can now be constructively applied in multicriteria decision making processes. Various possible extensions of this model and its potential applications allow us to expect more future research results devoted to anticipatory networks and their applications.

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